Warm-Up

CST/CAHSEE:AlgebraII
9.0/F-IF.7a,F-BF.3

Which of the following sentences is true about the graphs of $y=3(x-5)^2+1$ and $y=3(x+5)^2+1$?

A The vertices are maximums

B The graphs have the same shape with different vertices.

C The graphs have different shapes with different vertices.

D One graph has a vertex that is a maximum, while the other graph has a vertex that is a minimum.

- How might a student obtain each of the incorrect answers?

Review: AlgebraI
11.0,21.0/A-SSE.2a,F-IF.7a

Which are true statements of the function

$$f(x)=3x^2-6x-9$$

A) The vertex of the function is $(1,-12)$

B) 1 and -3 are roots of the function

C) The vertex of the function is a maximum

D) The graph of the function rises to the right and rises to the left

E) $(x+1)$ is a factor of the function

Current: Calculus 4.0

Power Rule for Derivatives

Given:
$$f(x)=x^n$$
$$f'(x)=nx^{n-1}$$

Use the power rule to find the derivative.

$$f(x)=x^3-6x^2+9x-1$$

Other: Algebra I 11.0,14.0/A-SSE.2a

Factor completely:

$$f(x)=10x^3-15x^2-20x$$

Warm-up Debrief:

Quadrant 1:

$f(x)=3x^2-6x-9$

$f(x)=3(x^2-2x-3)$

$=3(x-3)(x+1)$

$x-3=0 \quad x+1=0$

$x=3 \qquad x=-1$

$\therefore$ roots are $x=-1, 3$

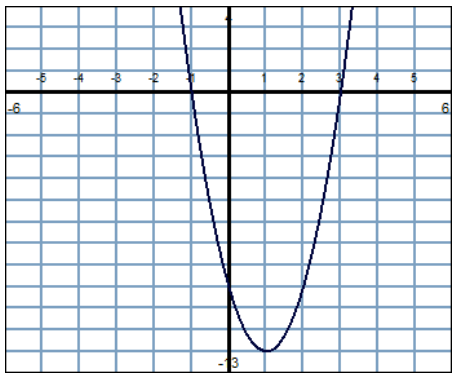
$f(x)=3x^2-6x-9$

$f(x)=3(x^2-2x)-9$

$=3(x^2-2x+(-1)^2)-9-3(-1)^2$

$=3(x-1)^2-12$

Vertex is $(1,-12)$



Vertex $(1,-12)$ is the minimum value.

The graph is concave up.

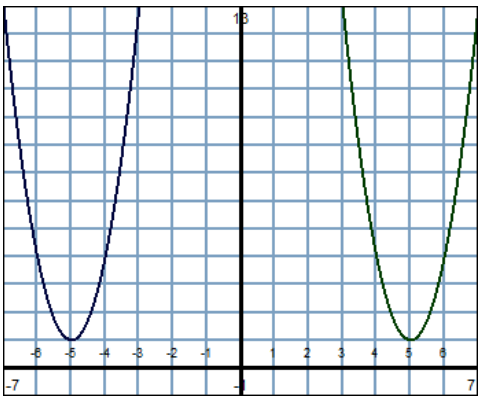
True statements are A, D, and E

Graph rises as $x \to \infty$ and as $x \to -\infty$

Quadrant 2:

Which of the following sentences is true about the graphs of $y=3(x-5)^2+1$ and $y=3(x+5)^2+1$?

Equations written in vertex form, $\therefore$ vertex of parabolas are $(5,1)$ and $(-5,1)$. Leading coefficient of both functions are positive, so graphs are concave up.



Correct answer is B

Quadrant 3:

Find the derivative:

$f(x)=x^3-6x^2+9x-1$

$f(x)=x^3-6x^2+9x^1-1x^0$

$f'(x)=3(x^{3-1})-2\bullet 6(x^{2-1})+1\bullet 9(x^{1-1})-1\bullet 0(x^{0-1})$

$f'(x)=3x^2-12x+9$

Quadrant 4:

Factor completely:

$f(x)=10x^3-35x^2-20x$

$=5x(2x^2-7x-4)$

$=5x(2x+1)(x-4)$

Connecting Algebra to Calculus
Particle Motion

Objective: Investigate the motion and velocity of a particle traveling along an *x*-axis and make connections to Algebra 1.

Standards: Algebra 1 14.0, 16.0, 21.0/F-IF.8a, F-IF.2, A-SSE.3a, Calculus 16.0

Lesson Notes: Print pages 15 – 17 for Example 2 "We Do" for students to work with you
Print pages 10 – 11 for the "You-Try!"

Lesson with Example #1

Consider a particle moving along an *x*-axis. The position of the particle on the *x*-axis is given by the **position function** $s(t) = t^3 - 6t^2 + 9t - 1,\ t \geq 0$, where time is measured in seconds.

The **velocity function**, $v(t)$, is the derivative of the position function, therefore $s'(t) = v(t)$.

The velocity function describes the speed of the particle and indicates the direction of the motion. Speed describes only how fast an object is moving, whereas velocity gives both how fast and in what direction the object is moving.

On an *x*-axis, the particle can only move to the left or to the right. If the velocity is positive, the particle is traveling to the right. If the velocity is negative, the particle is traveling to the left. In the case where the velocity is zero, the particle is stopped. We can summarize in the following chart.

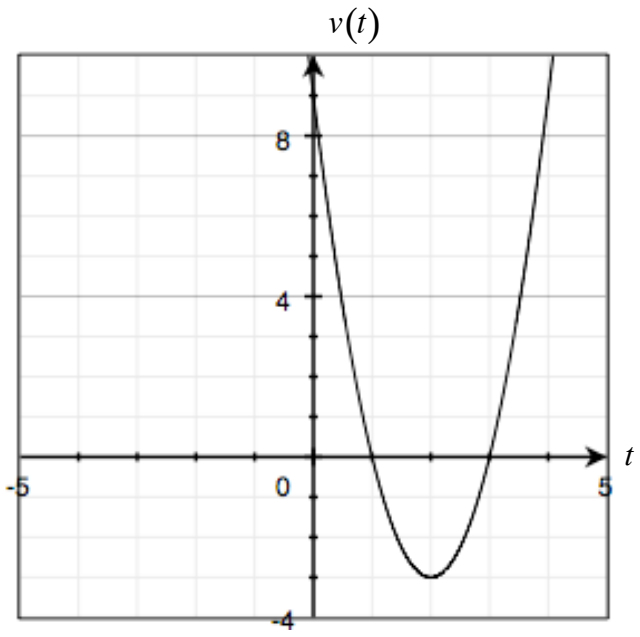
Velocity Function	Direction of Travel on an *x*-axis
$v(t) < 0$	Particle is traveling to the left
$v(t) = 0$	Particle is stopped
$v(t) > 0$	Particle is traveling to the right

Let's take the derivative of the position function, $s(t)$, to find the velocity function, $v(t)$.

$s(t) = t^3 - 6t^2 + 9t - 1$
$s'(t) = 3t^2 - 12t + 9$, therefore
$v(t) = 3t^2 - 12t + 9$

Graphing the velocity function will show us when the velocity is positive, negative or zero.

Notice that our velocity function is a quadratic function of *t*.
Finding the *t*-intercepts will help us draw the graph.



$$v(t) = 3t^2 - 12t + 9$$
$$3t^2 - 12t + 9 = 0$$
$$3(t^2 - 4t + 3) = 0$$
$$3(t-1)(t-3) = 0$$
$$(t-1)(t-3) = 0$$
$$t - 1 = 0 \text{ or } t - 3 = 0$$
$$t = 1 \text{ or } t = 3$$

These are the *t*-intercepts of our velocity graph.
Notice that axis of symmetry is the line $t = 2$,
Therefore the vertex has a *t*-coordinate of $t = 2$.
Substituting $t = 2$ in the velocity functions gives us

$$v(t) = 3t^2 - 12t + 9$$
$$v(2) = 3(2)^2 - 12(2) + 9$$
$$= 12 - 24 + 9$$
$$= -12 + 9$$
$$= -3$$

Therefore the vertex of the velocity function is $(2,-3)$.

To find the *y*-intercept:

$$v(0) = 3(0)^2 - 12(0) + 9$$
$$= 9$$

Therefore the coordinates of the *y*-intercept are $(0,9)$.

Let's investigate the motion of the particle from time $t = 0$ to $t = 4$ seconds.

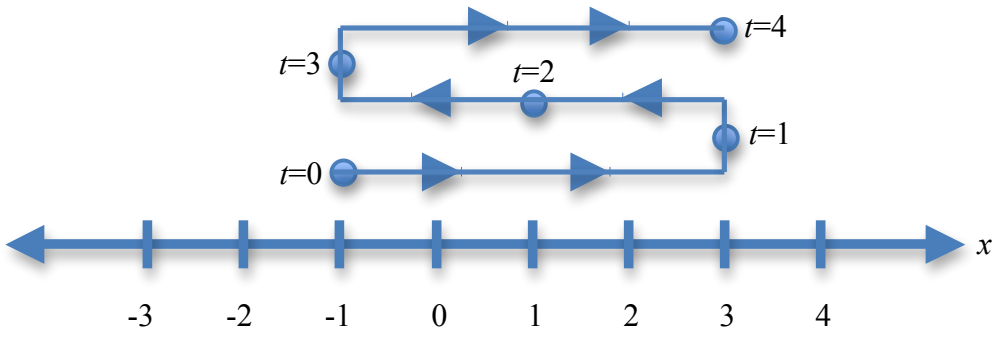
Velocity Function	Direction of Travel on an *x*-axis
$v(t) < 0$ on the interval $1 < t < 3$	Particle is traveling to the left
$v(t) = 0$ at $t = 1$ or $t = 3$	Particle is stopped
$v(t) > 0$ on the interval $0 \le t < 1$ and $3 < t \le 4$	Particle is traveling to the right

Now, let's investigate the position of the particle on the *x*-axis from time $t=0$ to $t=4$ seconds.

At time $t=0$, the position of the particle is $s(0)$, at time $t=1$, the position of the particle is $s(1)$ and so on.

Time (in seconds)	Position as a function of Time	Position on the *x*-axis
$t=0$	$s(0)=(0)^3-6(0)^2+9(0)-1$ $=-1$	At location $x=-1$
$t=1$	$s(1)=(1)^3-6(1)^2+9(1)-1$ $=3$	At location $x=3$
$t=2$	$s(2)=(2)^3-6(2)^2+9(2)-1$ $=1$	At location $x=1$
$t=3$	$s(3)=(3)^3-6(3)^2+9(3)-1$ $=-1$	At location $x=-1$
$t=4$	$s(4)=(4)^3-6(4)^2+9(4)-1$ $=3$	At location $x=3$

Here is a graph that shows the motion of the particle along an *x*-axis from time $t=0$ to $t=4$ seconds:



What is the Total Distance Traveled by the particle in the first 4 seconds?

(The particle traveled 12 units in the first 4 seconds.)

Displacement is the change in position from the initial position to the final position also indicating direction.

What is the Displacement of the particle over the first 4 seconds?

(The displacement of the particle over the first 4 seconds is 4 units to the right)

Answer the following questions and give a written justification for your answer:

1. When is the particle traveling to the left during the first 4 seconds? Justify

2. When is the particle traveling to the right during the first 4 seconds? Justify.

3. When is the particle stopped during the first 4 seconds? Justify.

4. Can you draw any conclusions regarding the motion of the particle after it has stopped? What is your conclusion?

Solutions:

1. The particle is traveling to the left on the time interval $1 < t < 3$ because $v(t) < 0$ on this time interval.

2. The particle is traveling to the right on the time intervals $0 \le t < 1$ and $3 < t \le 4$ because $v(t) > 0$ on these time intervals.

3. The particle is stopped at times $t = 1$ and $t = 3$ because $v(t) = 0$ at these times.

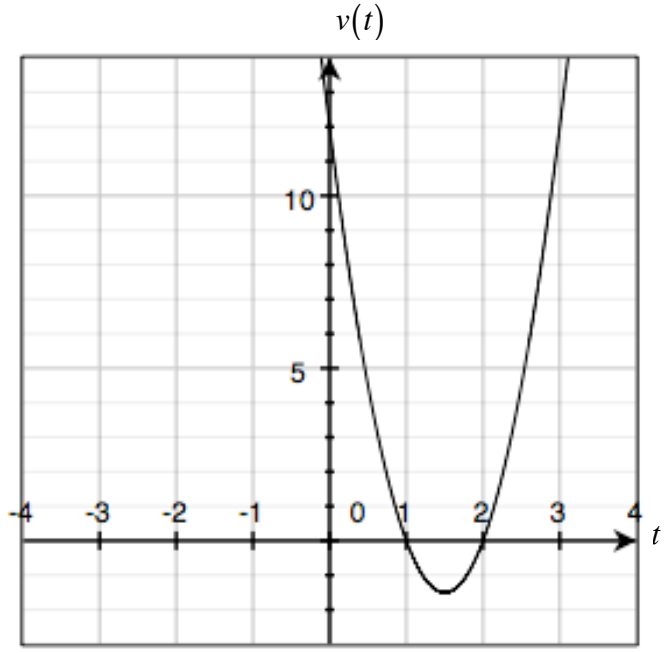
4. The conclusion I drew is that the particle has to stop before it can change direction.

Example #2 "We Do" (Note: Blank template at the end of the lesson for students to work with you)

Given the position function $s(t) = 2t^3 - 9t^2 + 12t - 4, t \geq 0$, for a particle moving along an *x*-axis, discuss the motion of the particle over the first 3 seconds the particle is traveling.

$s(t) = 2t^3 - 9t^2 + 12t - 4$

$v(t) = 6t^2 - 18t + 12$



$6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

$6(t-1)(t-2) = 0$

$t - 1 = 0$ or $t - 2 = 0$

$t = 1$ or $t = 2$

These are the *t*-intercepts of our velocity graph.

Notice that axis of symmetry is the line $t = \frac{3}{2}$,

Therefore the vertex has a *t*-coordinate of $t = \frac{3}{2}$.

Substituting $t = \frac{3}{2}$ in the velocity functions gives us

$$v\left(\frac{3}{2}\right) = 6\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 12$$

$$= 2 \bullet 3\left(\frac{9}{2 \bullet 2}\right) - \frac{54}{2} + \frac{12}{1}$$

$$= \frac{27}{2} - \frac{54}{2} + \frac{24}{2}$$

$$= -\frac{27}{2} + \frac{24}{2}$$

$$= -\frac{3}{2}$$

Therefore the vertex of the velocity function is

$$\left(\frac{3}{2}, -\frac{3}{2}\right) = (1.5, -1.5).$$

To find the *y*-intercept:

$$v(0) = 6(0)^2 - 18(0) + 12$$

$$= 12$$

Therefore the coordinates of the *y*-intercept are $(0,12)$.

Complete the table to investigate the velocity of the particle:

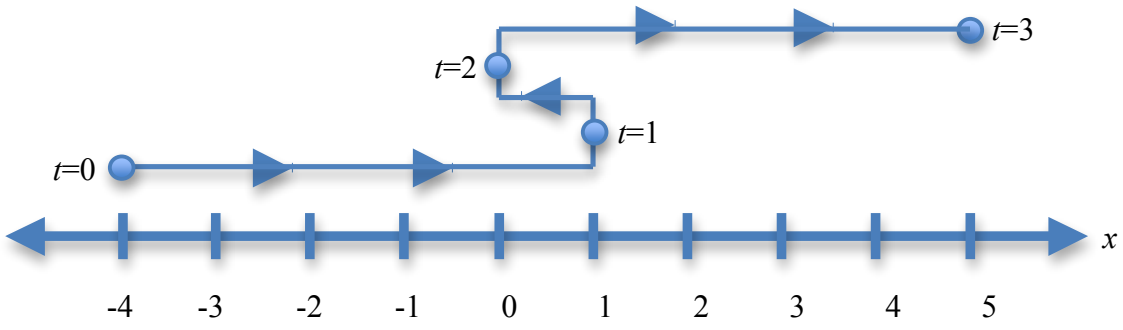
Velocity Function	Direction of Travel on an *x*-axis
$v(t) < 0$ on the interval $1 < t < 2$	Particle is traveling to the left
$v(t) = 0$ at $t = 1$ or $t = 2$	Particle is stopped
$v(t) > 0$ on the interval $0 \le t < 1$ and $2 < t \le 3$	Particle is traveling to the right

Complete the table to investigate the position of the particle on the *x*-axis:

Time (in seconds)	Position as a function of Time	Position on the *x*-axis
$t = 0$	$s(0) = 2(0)^3 - 9(0)^2 + 12(0) - 4$ $= -4$	At location $x = -4$
$t = 1$	$s(1) = 2(1)^3 - 9(1)^2 + 12(1) - 4$ $= 1$	At location $x = 1$
$t = 2$	$s(2) = 2(2)^3 - 9(2)^2 + 12(2) - 4$ $= 0$	At location $x = 0$
$t = 3$	$s(3) = 2(3)^3 - 9(3)^2 + 12(3) - 4$ $= 5$	At location $x = 5$

Draw a graph that shows the motion of the particle over the first 3 seconds:

Motion Graph



What is the Total Distance Traveled by the particle in the first 3 seconds?

(The particle traveled 11 units in the first 3 seconds.)

Displacement is the change in position from the initial position to the final position also indicating direction.

What is the Displacement of the particle over the first 3 seconds?

(The displacement of the particle over the first 3 seconds is 9 units to the right)

Answer the following questions and give a written justification for your answer:

1. When is the particle traveling to the left during the first 3 seconds? Justify

   (The particle is traveling to the left on the time interval $1 < t < 2$ because $v(t) < 0$ on this time interval.)

2. When is the particle traveling to the right during the first 3 seconds? Justify.

   (The particle is traveling to the right from time $0 \le t < 1$ and $2 < t \le 3$ because $v(t) > 0$ on these time intervals.)

3. When is the particle stopped during the first 3 seconds? Justify.

   (The particle is stopped at times $t = 1$ and $t = 2$ because $v(t) = 0$ at these times.)

You-Try! **Work in pairs or groups**

The position function $s(t) = -t^3 + 6t^2 - 9t + 1, t \geq 0$ gives the position of a particle moving along an *x*-axis. Investigate the motion of the particle over the first 5 seconds.

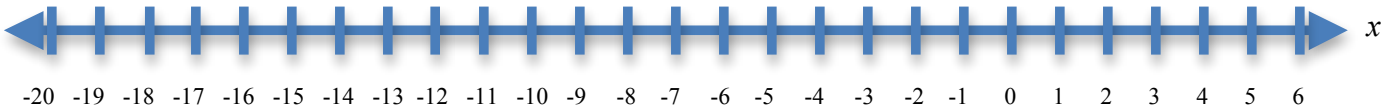
Complete the table to investigate the velocity of the particle:

Velocity Function	Direction of Travel on an *x*-axis

Complete the table to investigate the position of the particle on the *x*-axis:

Time (in seconds)	Position as a function of Time	Position on the *x*-axis

Motion Graph



What is the Total Distance Traveled by the particle in the first 5 seconds?

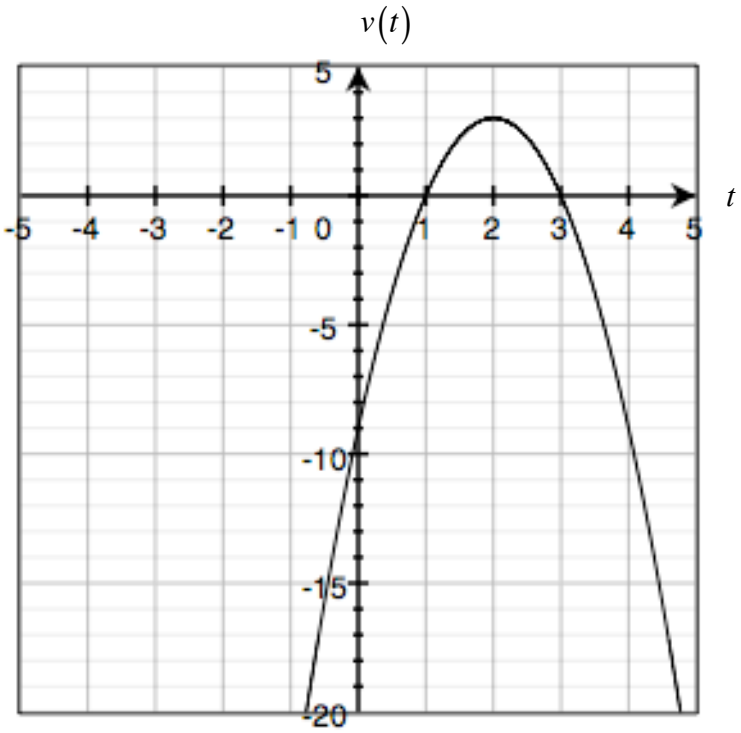
What is the Displacement of the particle over the first 5 seconds?

Displacement is the change in position from the initial position to the final position also indicating direction.

Answer the following questions and give a written justification for your answer:

1. When is the particle traveling to the left during the first 5 seconds? Justify
2. When is the particle traveling to the right during the first 5 seconds? Justify.
3. When is the particle stopped during the first 5 seconds? Justify.

Solutions to the You-Try!



$s(t) = -t^3 + 6t^2 - 9t + 1$

$v(t) = -3t^2 + 12t - 9$

$-3t^2 + 12t - 9 = 0$

$-3(t^2 - 4t + 3) = 0$

$-3(t-1)(t-3) = 0$

$t - 1 = 0$ or $t - 3 = 0$

$t = 1$ or $t = 3$

Axis of symmetry is the line $t = 2$

$v(2) = -3(2)^2 + 12(2) - 9$

$= -3(4) + 24 - 9$

$= -12 + 24 - 9$

$= 12 - 9$

$= 3$

Vertex has coordinates $(2,3)$

To Find the *y*-intercept:

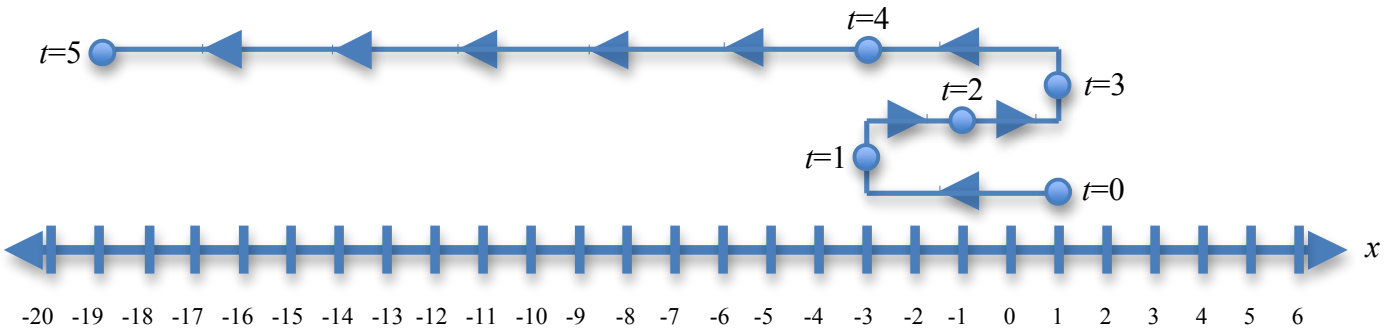
$v(0) = -3(0)^2 + 12(0) - 9$

$= -9$

Velocity Function	**Direction of Travel on an *x*-axis**
$v(t) < 0$ on the interval $0 \le t < 1$ and $3 < t \le 5$	Particle is traveling to the left
$v(t) = 0$ at $t = 1$ or $t = 3$	Particle is stopped
$v(t) > 0$ on the interval $1 < t < 3$	Particle is traveling to the right

Time (in seconds)	Position as a function of Time	Position on the x-axis
$t=0$	$s(0)=-(0)^3+6(0)^2-9(0)+1$ $=1$	At location $x=1$
$t=1$	$s(1)=-(1)^3+6(1)^2-9(1)+1$ $=-3$	At location $x=-3$
$t=2$	$s(2)=-(2)^3+6(2)^2-9(2)+1$ $=-1$	At location $x=-1$
$t=3$	$s(3)=-(3)^3+6(3)^2-9(3)+1$ $=1$	At location $x=1$
$t=4$	$s(4)=-(4)^3+6(4)^2-9(4)+1$ $=-3$	At location $x=-3$
$t=5$	$s(5)=-(5)^3+6(5)^2-9(5)+1$ $=-19$	At location $x=-19$

Motion Graph



What is the Total Distance Traveled by the particle in the first 5 seconds?

(The particle traveled 28 units in the first 5 seconds.)

What is the Displacement of the particle over the first 5 seconds?

(The displacement of the particle over the first 5 seconds is 20 units to the left)

Answer the following questions and give a written justification for your answer:

1. When is the particle traveling to the left during the first 5 seconds? Justify

   (The particle is traveling to the left on the time intervals $0 \le t < 1$ and $3 < t \le 5$ because $v(t) < 0$ on these time intervals.)

2. When is the particle traveling to the right during the first 5 seconds? Justify.

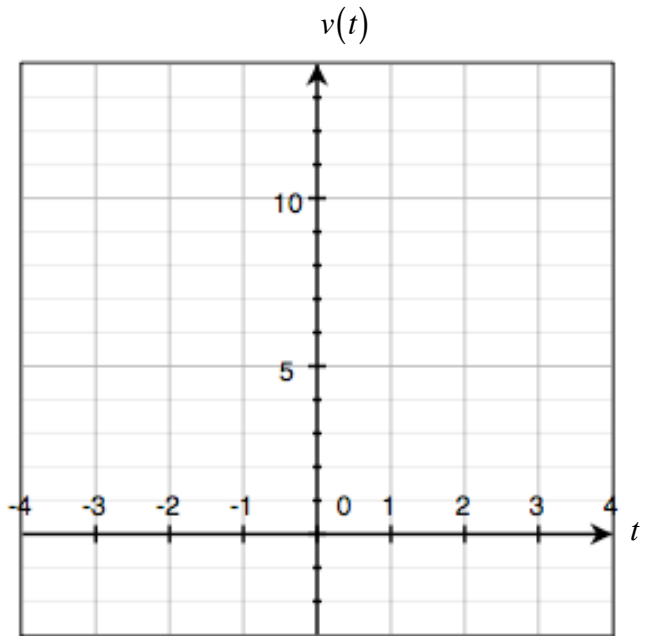
   (The particle is traveling to the right on the time interval $1 < t < 3$ because $v(t) > 0$ on this time interval.)

3. When is the particle stopped during the first 5 seconds? Justify.

   (The particle is stopped at times $t = 1$ and $t = 3$ because $v(t) = 0$ at these times.)

Example #2 (Blank template for students to work with you on Example 2 "We Do")

Given the position function $s(t) = 2t^3 - 9t^2 + 12t - 4, t \geq 0$, for a particle moving along an *x*-axis, discuss the motion of the particle over the first 3 seconds the particle is traveling.



The *t*-intercepts of our velocity graph are:

Notice that axis of symmetry is the line:

Therefore the vertex has a *t*-coordinate of:

Substituting in the velocity functions gives us:

Therefore the vertex of the velocity function is:

To find the *y*-intercept:

Therefore the coordinates of the *y*-intercept are:

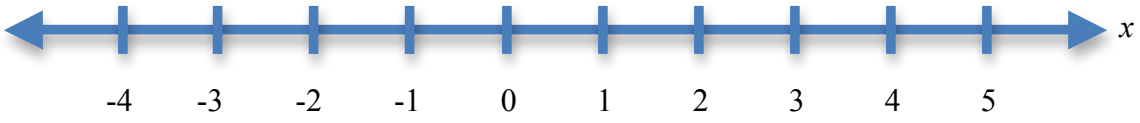
Complete the table to investigate the velocity of the particle:

Velocity Function	Direction of Travel on an *x*-axis

Complete the table to investigate the position of the particle on the *x*-axis:

Time (in seconds)	Position as a function of Time	Position on the *x*-axis

Draw a graph that shows the motion of the particle over the first 3 seconds:



What is the Total Distance Traveled by the particle in the first 3 seconds?

What is the Displacement of the particle over the first 3 seconds?

Displacement is the change in position from the initial position to the final position also indicating direction.

Answer the following questions and give a written justification for your answer:

1. When is the particle traveling to the left during the first 3 seconds? Justify

2. When is the particle traveling to the right during the first 3 seconds? Justify.

3. When is the particle stopped during the first 3 seconds? Justify.